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Dimensionless heat–mass transfer coefficients for forced convection around a sphere: a general low Reynolds number correlation

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Abstract—The average Nusselt number (Nu) for mass–heat transfer from a sphere is obtained as a function of the Peclet number (P) with the dimensionless Stokes drag F_0 ($2/3 \leqslant F_0 < 1$) as a parameter. The derivation considers in detail the high Peclet number range, where the Friedlander approximation might be employed. Considering a well-known Nu–P relation for bubbles ($F_0 = 2/3$) as the $F_0 = 2/3$ limit of a general $Nu = Nu(P, F_0)$ correlation allows us to explicitly obtain a rather simple expression of that relation for arbitrary F_0 ($2/3 \leqslant F_0 < 1$). The maximum error involved in using such a simple formula is estimated to be 6%.

1. INTRODUCTION

HEAT or mass transfer occuring between a continuous and a dispersed phase plays a major role in many processes of practical and industrial interest. Specifically, the rate of heat or mass transfer to or from spheres is fundamental to the analysis of chemical engineering operations, such as spray drying, extraction, humidification, aerosol scrubbing and evaporation. Even for spheres widely separated (on the average), so that interaction among them can safely be neglected the precise mathematical formulation of the transport problem is exceedingly difficult. Using dimensional reasoning it becomes apparent that the average Nusselt number, Nu, for that case will be a function of (a) σ, which denotes the Prandtl number in case of heat transfer or the Schmidt number in case of mass transfer, (b) Re, the Reynolds number of the flow and (c) any other parameters characterizing the two-phase system under consideration (e.g. viscosity ratio for liquid drops, slip-coefficients for rigid spheres with slip...). If the spheres are small enough ($< 100\,\mu$m) such that fluid inertia can be neglected, the correlation can only be of the form

$$Nu = Nu(P, F_0) \tag{1}$$

where P denotes the Peclet number ($P = Re\,\sigma$) and F_0 the (dimensionless) Stokes drag exerted upon the particle. ($F_0 = 1$ for a rigid sphere.) This general correlation has not been obtained, although various limiting forms have been derived.

1.1. *The low Peclet number limit, $P \ll 1$* [1,2]

$$Nu = 1+\tfrac{1}{2}P+\tfrac{1}{2}F_0P^2\ln P+g(F_0)P^2 +\tfrac{1}{4}F_0P^3\ln P+O(P^3) \tag{2}$$

with

$$g(F_0) = -\frac{13}{80}+\frac{1}{2}\left(\gamma+\frac{37}{120}\right)F_0+\frac{43}{320}F_0^2, \qquad \gamma = 0.5772156\ldots. \tag{3}$$

This result not only is rather complicated to derive but its convergence is very poor. As a matter of fact, up to the terms retained the range of validity of that relation is restricted to $P < 0.15$ [1].

1.2. *The high Peclet number limit, $P \gg 1$*

$$Nu = \begin{cases} 0.6246P^{1/3}[1+0.7381P^{-1/3}+\cdots], & F_0 = 1 \quad \text{(4a)} \\ 0.7979[(1-F_0)P]^{1/2}\left\{1+0.259\dfrac{2-F_0}{(1-F_0)^{3/2}}P^{-1/2}+\cdots\right\}, & F_0 \neq 1. \quad \text{(4b)} \end{cases}$$

The rigid sphere ($F_0 = 1$) result, equation (4a)—which is due to Acrivos and Goddard [3]—becomes valid for $P > 100$ [1]. On the other hand for spheres of general composition, Hirose's [4] result, equation (4b), requires $P \gg 0.067(2-F_0)^2/(1-F_0)^3$. For a droplet which is ten times as viscous as the suspending fluid ($F_0 = 0.97$) this requires Peclet numbers in excess of 10^4 and for more viscous droplets the restrictions on P become even more severe (recall $P = Re\ \sigma$ with $Re \ll 1$.) Besides that, equations (4a) and (4b) are restricted to the high Peclet number limit and equation (2) to the low Peclet number case. Consequently a wide gap of moderate Peclet numbers remains where the correlation (1) is not known. Equations (2), (4a) and (4b)

NOMENCLATURE

a	sphere radius	Greek symbols	
A_0, A_1	coefficients, defined by equations (24a) and (24b)	α	exponent appearing in the general correlation, equation (41) or equations (44a) and (44b)
b_i	functions of F_0 characterizing the general correlation, equation (11)	α_1, α_2	parameters, appearing in equation (22) and given by equation (36)
c	(dimensionless) concentration or temperature	β	slope of $Nu = Nu(P)$ curve for $P > 10^3$, equation (40)
C_R	retardation coefficient (according to the theory of Frumkin and Levich, equation (8b))	γ	Euler–Macheroni constant
		δ	(dimensionless) boundary layer thickness
D	mass or thermal diffusivity	δ_1	δ at leading edge
F_0	dimensionless Stokes drag	ε	(dimensionless) thickness of inside droplet in a multiple drop (encapsulated drop)
G_1, G_2	functions needed for encapsulated drops, equation (8e)		
$\mathbf{k}$	unit vector in the z-direction	ε_1	small parameter, needed for unnumerical computation (equations (39a) and (39b))
Nu	average Nusselt number		
P	Peclet number		
Q	volume flow rate of solute, equation (13)	ζ	normal coordinate inside boundary layer equation (18)
$\mathbf{r}$	position vector measured from sphere center: $r = \lvert\mathbf{r}\rvert$; $\partial/\partial\mathbf{r} = \nabla$, $\partial/\partial\mathbf{r}\cdot\partial/\partial\mathbf{r} = \nabla^2$	θ	polar angle
		$\kappa, \kappa_i, \kappa_M$	viscosity ratio (all relative to solvent viscosity)
r^*	distance of arbitrary point $\mathbf{r}^*$ outside the boundary layer	λ	slip coefficient, equation (8c)
		μ	$\cos\theta$
Re	Reynolds number (based on sphere radius)	σ	Schmidt or Prandtl number, P/Re
		τ	parameter, equation (42)
$\mathbf{v}$	local velocity	ϕ	stagnant cap angle, equation (8c)
v_∞	speed of approach.	ψ	Stokes stream function.

are plotted in Fig. 1 for the limiting cases $F_0 = 1$ (rigid sphere) and 2/3 (gas bubble).

Based on exact numerical solutions of the governing equation for bubbles and rigid spheres, respectively, Clift *et al.* [5] arrived at a general correlation of the form (1) for these two cases. It is the purpose of this paper to bridge the gap remaining and to obtain the general correlation (1) which reduces to Clift's result for $F_0 = 2/3$ and 1, respectively.

2. FORMULATION OF THE PROBLEM

Steady-state mass transfer from an isolated stationary sphere to an incompressible fluid medium (viscosity η) in laminar flow is governed by the equation

$$\nabla^2 c = P\mathbf{v}\cdot\frac{\partial}{\partial\mathbf{r}}c \tag{5}$$

subject to the boundary conditions*

$$c = 1, \quad \text{at} \quad r = 1 \tag{6a}$$

$$c = 0, \quad \text{as} \quad r \to \infty. \tag{6b}$$

Here, c is a normalized concentration, $\mathbf{v}$ the velocity vector scaled with the free stream velocity v_∞ (i.e. $\mathbf{v} = -\hat{\mathbf{k}}$ for $r \to \infty$), r the distance from the sphere center divided by the radius a, and $P = av_\infty/D$, a Peclet number based on the sphere radius (D is the mass diffusivity).

In the Stokes region, the stream function

$$\psi = \sin^2\theta\{\tfrac{1}{2}r^2 - \tfrac{3}{4}F_0 r - \tfrac{1}{2}(1-\tfrac{3}{2}F_0)r^{-1}\} \tag{7}$$

may be employed. Here, F_0 is the Stokes drag, made dimensionless by the factor $6\pi\eta a$ such that $F_0 = 1$ for a rigid sphere. Other cases which come to mind and

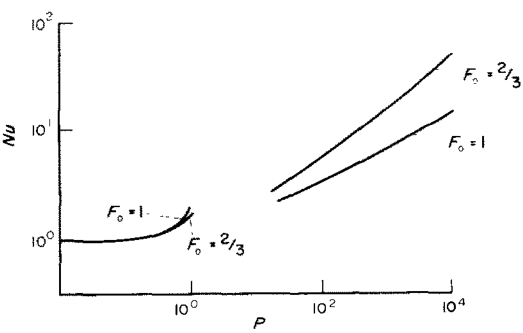

FIG. 1. The Nu–P correlation for the limiting cases $F_0 = 2/3$ and 1, respectively, according to the analytical results, equations (2), (4a) and (4b).

* We are assuming negligible internal resistance.

which (via F_0) are embraced by equation (7) are given below.

2.1. *Droplets* [6]

$$F_0 = \frac{\kappa+\frac{2}{3}}{\kappa+1} \tag{8a}$$

with κ the viscosity of the droplet phase relative to the fluid phase.

2.2. *Droplets with surface contamination*

If the contaminant is soluble in the continuous phase and if the (contaminant) concentration distribution over the interface is controlled by one of three rate limiting steps (i) adsorption–desorption kinetics, (ii) diffusion in the continuous phase or (iii) surface diffusion in the interface then, according to Frumkin and Levich [7]

$$F_0 = \frac{\kappa+\frac{2}{3}+C_R}{\kappa+1+C_R}. \tag{8b}$$

The retardation coefficient $C_R, 0 < C_R < \infty$, is different for the three cases. An equation formally identical to equation (8b) results also when the interfacial layer is treated as a liquid membrane. In this case the coefficient C_R is equal to two thirds of the membrane viscosity divided by the drop radius and by the solvent viscosity [8].

2.3. *Stagnant cap*

The Frumkin–Levich approach predicts a symmetrical internal circulation. Experimental observations have shown that the surfactant often collects in the form of a stagnant cap at the rear of the drop [5]. If ϕ denotes the angle of this (immobile) region, then F_0 is given by [9]

$$F_0 = \frac{\kappa+\frac{2}{3}}{\kappa+1} + \frac{2\phi+\sin\phi-\sin 2\phi-\frac{1}{3}\sin 3\phi}{6\pi(1+\kappa)}. \tag{8c}$$

2.4. *Spheres with slip (which are encountered in gases when the sphere radius is approximately* 10–100 *mean free path lengths)*

$$F_0 = \frac{1+2\lambda}{1+3\lambda} \tag{8d}$$

with λ the dimensionless slip coefficient [10].

2.5. *Encapsulated droplets* [11]

In this case a liquid shell of thickness $(1-\varepsilon)$ and viscosity κ_M (relative to the solvent) completely encapsulates a droplet of radius ε and relative viscosity κ_i. For this (type A) multiple emulsion drop, is given by ref. [11]

$$F_0 = \frac{2}{3}\frac{\kappa_i+\kappa_M(2+3\kappa_i)G_1(\varepsilon)+6\kappa_M^2G_2(\varepsilon)}{\kappa_i+2\kappa_M(1+\kappa_i)G_1(\varepsilon)+4\kappa_M^2G_2(\varepsilon)} \tag{8e}$$

with

$$G_1(\varepsilon) = \frac{(1+\varepsilon)(2+\varepsilon+2\varepsilon^2)}{(1-\varepsilon)(4+7\varepsilon+4\varepsilon^2)}$$

$$G_2(\varepsilon) = \frac{1-\varepsilon^5}{(1-\varepsilon)^3(4+7\varepsilon+4\varepsilon^2)}.$$

It is interesting to see that in all cases, F_0 varies between 2/3 and 1. (As far as equation (8e) is concerned, the corresponding proof is somewhat involved.)

Of primary interest is the average mass transfer from the sphere, i.e. the Nusselt number

$$Nu = \frac{1}{2}\int_0^\pi d\theta \sin\theta \left.\frac{\partial c}{\partial r}\right|_{r=1}. \tag{9}$$

This quantity has been obtained for $P \ll 1$ by singular perturbation techniques (equation (2)) and for $P \gg 1$ by means of the boundary layer concept (equations (4a) and (4b). For intermediate values of P, the exact numerical results obtained for $F_0 = 2/3$ and 1, respectively, are known to be approximated extremely well by the following relations [5]

$$Nu = \tfrac{1}{2}+\tfrac{1}{2}(1+2P)^{1/3}, \quad F_0 = 1 \tag{10a}$$

$$Nu = \tfrac{1}{2}+\tfrac{1}{2}(1+0.895P^{2/3})^{3/4}, \quad F_0 = \tfrac{2}{3}. \tag{10b}$$

These results are of the general form

$$Nu = 1/2+1/2(1+b_1P^{b_2})^{b_3} \tag{11}$$

with $b_i = b_i(F_0)$, $i = 1, 2, 3$.

In order to obtain the functions b_i over the whole range of $F_0(2/3 \leqslant F_0 < 1)$, we utilize the Friedlander approximation [12]. Although not exact, this method has been recommended for predicting the Nu–P relationship for arbitrary F_0 [5]. The approximation itself consists in the neglect of molecular diffusion in the θ direction relative to diffusion in the normal direction. Assuming that a concentration boundary layer of thickness $\delta = \delta(\theta)$ (which need not be small) exists such that c has dropped to its free stream value of zero outside this layer, a partial integration of equation (5) furnishes

$$\frac{d}{d\theta}Q = -\frac{2\pi}{P}\sin\theta\left.\frac{\partial c}{\partial r}\right|_{r=1} \tag{12}$$

with

$$Q = 2\pi\int_1^{r^*} dr\, c\frac{\partial}{\partial r}\psi, \quad r^* \geqslant 1+\delta \tag{13}$$

the amount of solute passing the cone shaped surface θ = const. Thus we have

$$Q(0) = 0 \tag{14}$$

and integration of equation (12) furnishes

$$Nu = \frac{P}{4\pi}Q(\pi). \tag{15}$$

Quite clearly, the neglect of azimuthal diffusion is justified for $P \to 0$ (pure diffusion) and for $P \gg 1$. For $P = 0$ (where $\delta = \infty$), the concentration is given by

$c = 1/r$ and equation (12) implies

$$\lim_{P\to 0}(PQ) = 2\pi(1-\cos\theta). \quad (16)$$

The resulting limit of one for Nu is well established. At the other extreme (i.e. for $P \gg 1$), the concentration boundary layer thickness δ will be small, at least close to the forward stagnation point, where most heat–mass transfer will take place. Thus, as far as Q is concerned, the behavior of ψ close to the sphere surface is all that matters. Using equation (7), we find

$$\psi = \tfrac{3}{2}\sin^2\theta\zeta\delta[(1-F_0)+\tfrac{1}{2}F_0\zeta\delta+O(\delta^2)] \quad (17)$$

where we have put

$$r = 1+\zeta\delta. \quad (18)$$

This being the case, Q becomes

$$Q = 3\pi\sin^2\theta\delta\int_0^1 d\zeta[(1-F_0)+F_0\zeta\delta+\cdots]c. \quad (19)$$

Although the evaluation of that integral requires the detailed knowledge of the concentration, we will employ a simple polynomial approximation for c. This approach is entirely analogous to the corresponding one for the momentum integral method (Karman–Pohlhausen) where it is well established that even extremely simple approximations for the velocity field within the viscous boundary layer lead to extremely good results for average bulk quantities.

In our case, the physically obvious boundary conditions are

$$\zeta = 0: \quad c = 1, \quad \frac{\partial c}{\partial \theta} = 0 \quad (20a, b)$$

$$\zeta = 1: \quad c = 0, \quad \frac{\partial c}{\partial \zeta} = 0. \quad (20c, d)$$

If equation (5) is applied at the sphere surface, then the boundary condition (20b) requires

$$\frac{\partial^2 c}{\partial \zeta^2} = 0 \quad \text{at} \quad \zeta = 0. \quad (21)$$

For reasons soon to become apparent, we choose the fifth-order polynomial

$$c = (1-\zeta)^2\{(1+\tfrac{1}{2}\zeta)+\alpha_1\zeta(\tfrac{1}{2}+\zeta)+\alpha_2\zeta(1+\zeta)^2\} \quad (22)$$

which for arbitrary coefficients α_1 and α_2, satisfies all the boundary conditions listed. As far as equation (19) is concerned, we thus get

$$Q = \delta\sin^2\theta[A_0+A_1\delta] \quad (23)$$

with

$$A_0 = \frac{9\pi}{8}(1-F_0)\left[1+\frac{\alpha_1}{5}+\frac{4}{9}\alpha_2\right] \quad (24a)$$

and

$$A_1 = \frac{3\pi}{10}F_0\left[1+\frac{\alpha_1}{3}+\frac{16}{21}\alpha_2\right] \quad (24b)$$

two purely numerical coefficients (for fixed F_0).

Solving equation (23) for δ furnishes $\delta = \delta(Q)$

$$\delta = \frac{1}{2A_1}\left\{\left[A_0^2+\psi\frac{A_1 Q}{1-\mu^2}\right]^{1/2}-A_0\right\}, \quad \mu = \cos\theta \quad (25)$$

and with this expression and c given by equation (22), equation (12) becomes

$$\frac{d}{d\mu}Q = -\frac{3\pi}{P}\left(1-\frac{\alpha_1}{3}-\frac{2}{3}\alpha_2\right)\delta^{-1} \quad (26)$$

with

$$Q(1) = 0. \quad (27)$$

An analytical solution of that equation is possible for the asymptotic limit $P \to \infty$. Depending on whether A_0 vanishes or not two cases have to be distinguished.

(a) Rigid spheres, $F_0 = 1$ (i.e.

$$A_0 = 0, A_1 = 3\pi/10[1+\alpha_1/3+16/21\alpha_2]).$$

In this case

$$\delta = A_1^{-1/2}\left(\frac{Q}{1-\mu^2}\right)^{1/2} \quad (28)$$

and so

$$Q^{3/2} = \frac{9\pi}{4P}A_1^{1/2}\left(1-\frac{\alpha_1}{3}-\frac{2}{3}\alpha_2\right) \times\left[\frac{\pi}{2}-\sin^{-1}\mu-\mu\sqrt{(1-\mu^2)}\right]. \quad (29)$$

This implies

$$Nu = 0.616\left[\left(1+\frac{\alpha_1}{3}+\frac{16}{21}\alpha_2\right) \times\left(1-\frac{\alpha_1}{3}-\frac{2}{3}\alpha_2\right)^2\right]^{1/3}P^{1/3}. \quad (30)$$

According to equation (10a) we should get

$$Nu = 0.630P^{1/3}. \quad (31)$$

(b) $2/3 \leqslant F_0 \leqslant 1$. In this case neither A_0 nor A_1 vanish. For $P \to \infty$ we expect that $\delta \to 0$ close to the leading edge. This then implies that most of the transfer will take place close to the leading edge. Assuming A_0 and A_1 to be bounded, there exists some range of θ—close to the leading edge—where the condition $A_1\delta \ll A_0$ will be met. Within this θ-range, δ is given by

$$\delta = \frac{1}{A_0}\frac{Q}{1-\mu^2} \quad (32)$$

and consequently, by equations (26) and (27)

$$Q^2 = \frac{4\pi}{P}A_0\left(1-\frac{\alpha_1}{3}-\frac{2}{3}\alpha_2\right)(1-\mu)^2\left(1+\frac{1}{2}\mu\right). \quad (33)$$

The resulting Nusselt number is

$$Nu = \frac{3}{4}(1-F_0)^{1/2}\left[\left(1-\frac{\alpha_1}{3}-\frac{2}{3}\alpha_2\right) \times\left(1+\frac{\alpha_1}{5}+\frac{4}{9}\alpha_2\right)\right]^{1/2}P^{1/2}. \quad (34)$$

According to equation (10b), we should get for $F_0 = 2/3$

$$Nu = 0.460P^{1/2}. \tag{35}$$

If we choose*

$$\alpha_1 = -20.569, \quad \alpha_2 = 10.772 \tag{36}$$

then, according to equations (30) and (34), respectively

$$Nu = \begin{cases} 0.630P^{1/3}, & F_0 = 1 \quad (37a) \\ 0.797\sqrt{(1-F_0)}P^{1/2}, & \frac{2}{3} \leqslant F_0 < 1 \quad (37b) \end{cases}$$

in perfect agreement with equations (31) and (35), respectively. Thus, we shall assume that the use of equation (36) in equation (22) will, by equation (26), produce the correct $P \gg 1$ behavior of the Nu–P relation over the whole range of F_0.

Since the rigid sphere case $F_0 = 1$ is seen to be singular (for $P \gg 1$) we shall not include it in any general considerations to follow. By equation (10a) the $Nu = Nu(P)$ relation is known for that case anyhow. For equation (37b) to be valid the relation $A_1\delta \ll A_0$ has to be met close to $\mu = 1$. Using equation (33) in equation (32) we get δ proportional to $[P(1-F_0)]^{-1/2}$ close to the leading edge. Consequently the restriction $A_1\delta \ll A_0$ translates into the condition $P \gg F_0^2/(1-F_0)^3$ for validity of equation (37b)† The asymptotic high Peclet number limit, equation (37b), requires larger and larger Peclet numbers the closer F_0 gets to one. For example: for $F_0 = 0.97$ (which for droplets corresponds to a viscosity ratio $\kappa = (10)$ the Peclet number would have to be larger than 10^5. It is doubtful that for the low Reynolds number case under consideration such high Peclet numbers can be realized in practice. In order to see what happens for moderate Peclet numbers we turn to some numerical results.

3. NUMERICAL METHOD AND RESULTS

If for $P \gg 1$ the restriction $P \gg F_0^2/(1-F_0)^3$ is not met equation (26) has to be solved numerically from $\mu = 1$ to -1. In order to use equation (25), we have to realize that the term $Q/(1-\mu^2)$ becomes indeterminate as $\mu \to 1$. The first step was to determine $\delta_1 (\equiv \delta(\mu = 1))$ from the relation‡

$$\delta_1 = \frac{1}{2A_1}\left[\left\{A_0^2 + \frac{4.05\pi}{P}A_1\delta_1^{-1}\right\}^{1/2} - A_0\right]. \tag{38}$$

Although all three roots of this cubic equation for δ_1 are real (for $A_0 > 0$ and $A_1 > 0$), two of them are negative and the evaluation of the only positive root was readily achieved via a Newton–Raphson iteration. With δ_1 known, equation (26) was restricted to the interval $-1 < \mu < 1-\varepsilon_1, \varepsilon_1 \ll 1$

$$\frac{\mathrm{d}}{\mathrm{d}\mu}Q = -\frac{2.025\pi}{P}\delta^{-1} \tag{39a}$$

subject to the boundary condition

$$Q(1-\varepsilon_1) = \frac{2.025\pi}{P\delta_1}\varepsilon_1. \tag{39b}$$

It turned out that $\varepsilon_1 = 0.001$ was small enough such that a decrease of ε below this value left the first three digits of the result unaffected. A fourth-order Runga–Kutta numerical method was employed to solve differential equations (39a) and (39b). For $10^3 < \mathrm{P} < 10^4$ the resulting Nusselt number is plotted in Fig. 2.

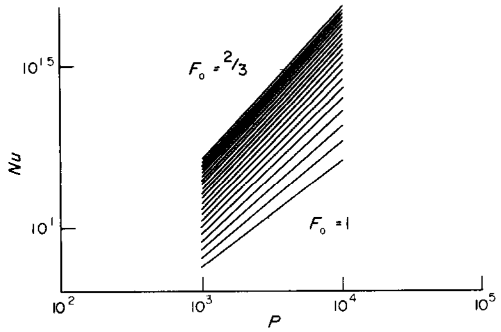

FIG. 2. The Nu–P correlation for various F_0. Successive lines differ in their F_0 value by 1/60.

Increasing F_0 between 2/3 and 1 by steps of 1/60 leads to the 21 curves shown. As can clearly be seen the change from the 1/3 slope (for $F_0 = 1$) to the 1/2 slope (for $F_0 = 2/3$) is gradual with the biggest changes occurring for F_0 close to one. This implies that, as far as droplets are concerned, small amounts of contamination which tend to suppress the internal circulation (and thus push F_0 close to one, see equation (8b)) can rather drastically reduce the mass transfer. At the other extreme, i.e. for F_0 close to 2/3 the slope changes very little from its limit of 1/2. This implies that the 1/2 slope, correct for bubbles ($\kappa = 0$) is also a good approximation for drops of low viscosity. Experimentally this has been known for some time [13].

Repeating the calculation for larger Peclet numbers exemplifies this trend, i.e. a far more rapid change close to $F_0 = 1$ and hardly any change at all in the slope for increasingly larger ranges of F_0 at the other end of the spectrum. Formally putting

$$Nu \propto P^\beta, \quad \text{for} \quad P \gg 1 \tag{40}$$

the slope β is plotted in Fig. 2 with $P = 10^m$, $m = 3, \ldots, 10$ as parameter. The singularity of the rigid sphere limit ($F_0 = 1$) becomes quite apparent from this graph.

In order to obtain for $2/3 \leqslant F_0 < 1$ a general correlation of the form (11) we make use of the asymptotic limit (37b). With $b_3 = 1/\alpha$ we are thus led to

* Note that this choice implies $A_0 = 1.883\pi(1-F_0)$, $A_1 = 0.705\pi F_0$ and $(\mathrm{d}/\mathrm{d}\mu)Q = -(2.025\pi/P)\delta^{-1}$.

† This restriction has been obtained before by entirely different arguments [2].

‡ To obtain that relation we utilize in equation (25) the expansion $Q(\mu) \cong -(1-\mu)(\mathrm{d}/\mathrm{d}\mu)Q$ for μ close to one.

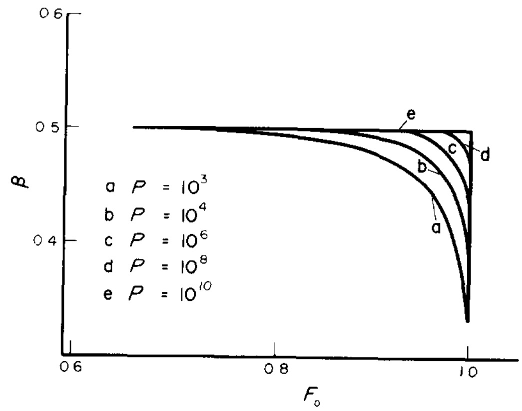

Fig. 3. The exponent β of the $Nu = CP^{\beta}$ correlation in the high Peclet number limit.

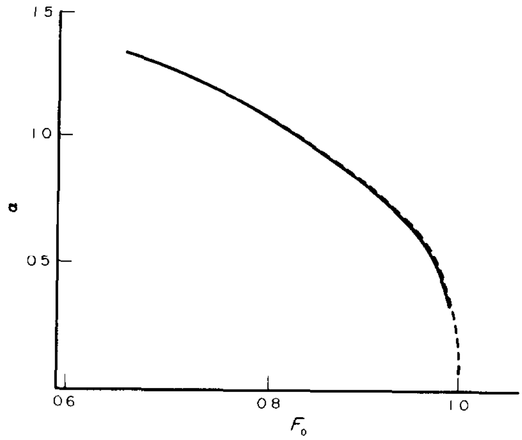

Fig. 4. The function $\alpha = \alpha(F_0)$ according to equation (41): ----, numerical.

the relation

$$Nu = \tfrac{1}{2}+\tfrac{1}{2}[1+(1.594\sqrt{(1-F_0)}P^{1/2})^{\alpha}]^{1/\alpha}, \quad \text{for } \tfrac{2}{3} \leqslant F_0 < 1. \quad (41)$$

By equation (10b) we know $\alpha = 4/3$ for $F_0 = 2/3$. In order to obtain the function $\alpha = \alpha(F_0)$ over the whole range $2/3 \leqslant F_0 < 1$ we utilize our previous numerical results for Nu at $P = 10^3$.* With Nu computed, equation (41) can be solved for α. The dashed line in Fig. 4 shows the result. Although $\alpha = \alpha(F_0)$ is defined only for $2/3 \leqslant F_0 < 1$ the limit $\alpha \to 0$ for $F_0 \to 1$ is ultimately approached with an infinite slope. To reproduce that trend we try the approximation

$$\alpha = \tfrac{4}{3}[3(1-F_0)]^{\tau}, \quad \tau < 1. \quad (42)$$

To determine τ we minimized the mean square deviation, using a numerical package called MLAB (Modelling Laboratory), a general purpose DEC system computer program. With an RMS error of about 3%, τ turned out to be 0.395. This implies the use of

$$\alpha = 2.058(1-F_0)^{0.395} \quad (43)$$

in equation (41). The resulting $Nu = Nu(P \cdot F_0)$ correlation has been plotted in Fig. 5 for $F_0 = 2/3$, 0.778, 0.889, 0.960 and 0.970 (For completeness equation (10a) for $F_0 = 1$ has also been included in this graph. These values of F_0 have been chosen in order that we can compare with reported results (Fig. (3.10) of Clift *et al.* [5]). As far as we can tell by comparing these graphs the results match perfectly.

4. SUMMARY AND CONCLUSION

In this paper we focused attention on the average heat–mass transfer from a sphere under creeping motion conditions. Assuming that the Friedlander approximation is valid for $P > 10^3$ led us to our main formula

$$Nu = 1/2+1/2[1+(1.594\sqrt{(1-F_0)}P^{1/2})^{\alpha}]^{1/\alpha}, \quad 2/3 \leqslant F_0 < 1 \quad (44a)$$

with

$$\alpha = 2.058(1-F_0)^{0.395}. \quad (44b)$$

In conjunction with the corresponding formula (10a) for the exceptional case $F_0 = 1$ we thus have an expression for the general correlation (1). Though simple in form, the error involved in this correlation remains to be determined. Comparing with exact numerical results, Clift *et al.* [5] estimated that for $F_0 = 1$, equation (10a) agrees with the numerical solution within 2%. For the other limiting case ($F_0 = 2/3$) Clift's error estimate of equation (10b)—or equations (44a) and (44b)—is 6% for all P. For intermediate F_0 we turn to the numerical results of Wellek *et al.* [14]. They used $F_0 = 0.781$, a value which is attained for a droplet of ethyl acetate rising through water ($\kappa = 0.52$). If we assume $Re < 0.1$ to properly represent low Reynolds number predictions then $P < 5 \times 10^3$ seems to be an absolute upper bound for all fluids of practical interest. Within the range $0 < P < 5 \times 10^3$ the graphically reported numerical results for $F_0 = 0.781$ [14] differ

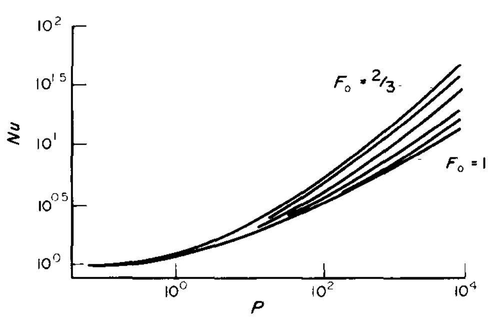

Fig. 5. The $Nu = Nu(P, F_0)$ correlation according to equations (41) and (43).

* Recall the high Peclet number restriction $P \gg F_0^2/(1-F_0)^3$ for validity of the asymptotic result (37b).

from predictions according to equations (44a) and (44b) by less than 4%. Assuming this to be somehow characteristic for all intermediate values of F_0 (i.e. $2/3 \leqslant F_0 < 1$) the absolute error estimate of equations (44a) and (44b) will be 6% for all $F_0 < 1$. Being rather accurate yet simple in form, equations (11a), (44a) and (44b) should thus prove useful for practical purposes. How these correlations are modified for non-zero Reynolds numbers has to await further study.

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COEFFICIENTS ADIMENSIONNELS DE TRANSFERT DE CHALEUR ET DE MASSE POUR LA CONVECTION FORCEE AUTOUR D'UNE SPHERE: UNE FORMULE POUR UN FAIBLE NOMBRE DE REYNOLDS

Résumé—Le nombre de Nusselt moyen (Nu) pour le transfert de masse et de chaleur à partir d'une sphère est obtenu en fonction du nombre de Péclet (P) avec la trainée adimensionnelle de Stokes F_0 ($2/3 \leqslant F_0 < 1$) comme paramètre. La dérivation considère en détail le domaine des grands nombres de Péclet où l'approximation de Friedlander peut être employée. Considérant une relation Nu–P bien connue pour les bulles ($F_0 = 2/3$) comme la limite $F_0 = 2/3$ d'une relation générale $Nu = Nu(P, F_0)$, on obtient explicitement une expression plutôt simple de cette relation pour F_0 arbitraire ($2/3 \leqslant F_0 < 1$). L'erreur maximale en utilisant cette formule simple est estimée à 6%.

DIMENSIONSLOSE WÄRME- UND STOFFÜBERGANGSKOEFFIZIENTEN FÜR DIE ERZWUNGENE KONVEKTION UM EINE KUGEL: EINE VERALLGEMEINERTE KORRELATION FÜR KLEINE REYNOLDS-ZAHLEN

Zusammenfassung—Die mittlere Nusselt-Zahl (Nu) für den Wärme- und Stoffübergang an einer Kugel ergibt sich als Funktion der Peclet-Zahl (P) in Abhängigkeit vom dimensionslosen Stokes-Widerstands-Parameter F_0 ($2/3 \leqslant F_0 < 1$). Die Herleitung berücksichtigt insbesondere den Bereich großer Peclet-Zahlen, in dem die Friedlander-Approximation angewandt werden darf. Die Verwendung einer bekannten Nu–P-Beziehung für Blasen ($F_0 = 2/3$) als Grenzfall der allgemeinen Beziehung $Nu = Nu(P, F_0)$ ergibt eine ziemlich einfache Formulierung jener Beziehung für beliebige F_0 ($2/3 \leqslant F_0 < 1$). Der maximale Fehler, der beim Anwenden dieser einfachen Formel auftritt, wird zu 6% veranschlagt.

БЕЗРАЗМЕРНЫЕ КОЭФФИЦИЕНТЫ ТЕПЛО-И МАССОПЕРЕНОСА ПРИ ВЫНУЖДЕННОЙ КОНВЕКЦИИ ВОКРУГ СФЕРЫ. ВЫВОД ОБОБЩЕННОЙ ЗАВИСИМОСТИ В СЛУЧАЕ МАЛЫХ ЗНАЧЕНИЙ ЧИСЛА РЕЙНОЛЬДСА

Аннотация—Дан вывод среднего значения числа Нуссельта (Nu) для массопереноса от сферы как функции числа Пекле (P), где в качестве параметра используется безразмерный стоксовский коэффициент сопротивления $F_0 = (2/3 \leq F_0 < 1)$. Рассматривается интервал больших значений числа Пекле, в котором можно применить приближение Фридлендера. С помощью известного соотношения Nu–P для пузырьков ($F_0 = 2/3$), которое является пределом $F_0 = 2/3$ для общей зависимости $Nu = Nu(P, F_0)$, можно получить довольно простое выражение этого соотношения в явном виде для произвольных значений F_0 ($2/3 \leq F_0 < 1$). Показано, что использование такой простой формулы дает максимальную погрешность в 6%.